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MINIMUM BUBBLE DEPARTURE DIAMETER IN NUCLEATE POOL BOILING

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Abstract—A method is described for determining the minimum bubble departure diameter in saturated nucleate pool boiling. It is shown that the minimum departure size varies with Jakob number only. Predicted results are compared with the smallest experimental departure diameters reported in literature. Satisfactory agreement is achieved when the minimum departure size has been obtained on the basis of bubble growth rate law recommended by Cole and Shulman.

NOMENCLATURE

a, b, c, coefficients appearing in equation (8);
c_L, specific heat of liquid;
D, instantaneous bubble diameter;
D_b, bubble base diameter;
D_d, departure diameter;
D_{min}, minimum departure diameter;
$\bar{D}_{min}$, dimensionless minimum departure diameter; defined by equation (21);
$f_i(\phi)$, function, defined by equation (12);
F_b, buoyant force;
F_{sy}, surface tension force; vertical component;
F_L, liquid inertia force;
F_R, pressure restraining force; given by equation (7);
g, gravitational acceleration;
g_0, conversion constant;
$G(\theta)$, defined by equation (4);
N_{Ja}, Jakob number $= \Delta T c_L \rho_L / \rho_v \lambda$ [dimensionless];
t, time;
ΔT, difference between wall temperature and saturation temperature.

Greek symbols
α, thermal diffusivity;
β, growth constant, defined by equation (1);
γ, angle, shown in Fig. 1;
θ, bubble contact angle;
λ, latent heat of vaporization;
ρ_L, ρ_v, liquid and vapor densities, respectively,
σ, surface tension;
ϕ, parameter, defined by equation (13).

1. INTRODUCTION

IN THE nucleate pool boiling the bubble departure process is controlled by large numbers of variables. A complete analysis of the problem is not available although several expressions have been proposed for predicting the departure size [1]. One major difficulty concerning this problem is that the boundary conditions at the heated wall are uncertain. Therefore, the question may be raised if it is possible to acquire any information pertinent to the bubble departure without knowing in advance the boundary conditions at the heated wall. The present investigation indicates that this is possible when the objective of the analysis is to determine the minimum bubble departure diameter.

It is shown that the minimum bubble size can be expressed, in terms of Jakob number, by a simple equation. Derivation of this equation assumes that the bubble growth rate law is already known. A comparison between the predicted results and the experimental data has been made for Jakob numbers ranging from 10 through 792. This study, besides providing the minimum departure diameter, helps one to identify the relevant thermodynamic parameters which influence the departure phenomenon.

2. ANALYSIS

Following are the fundamental assumptions used in the analysis:

(i) Vapor bubbles are assumed to be isolated. Bulk of the liquid is at the saturation temperature.

(ii) The bubble is considered to be of spherical form ending to a small neck which connects the bubble to the heated surface, Fig. 1.

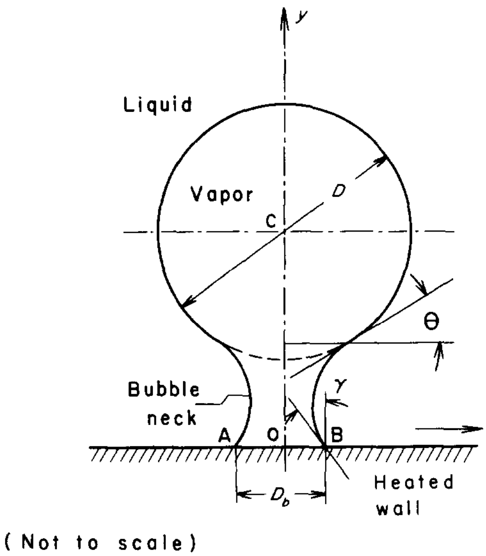

FIG. 1. Geometry of the growing bubble.

Johnson *et al.* [2] have reported that, among other bubble shapes, the spherical bubble has the smallest departure size. Since the present analysis is concerned with the minimum departure size, assumption of the spherical bubble shape is plausible.

(iii) Forces acting on the isolated bubble have been computed by following the method of Keshock and Siegel [3] with the exception of a so-called pressure restraining force which will be discussed later. In agreement with the experimental results reported in [2], the drag force and the vapor inertia force are assumed to be negligible in comparison with the other forces.

(iv) The contact angle θ is defined as shown in Fig. 1. This requires modification of the surface tension force formulated in [3]. Relation given in this reference does not take into account presence of the bubble neck.

(v) The bubble growth parameter β is given by

$$D = \beta t^{\frac{1}{2}}. \tag{1}$$

(vi) Departure of the bubble from the heating surface is described by application of the force balance equation just prior to the necking down of the bubble. In the earlier stages of growth, this equation is not satisfied and the vapor bubble is held down on the surface by hydrodynamic forces. It is assumed that the bubble departure from the surface will not occur until the force balance equation is satisfied.

Referring to Fig. 1, forces acting on the growing bubble are given by the following relations:

Buoyant force

$$F_b = \frac{\pi}{6} g \frac{\rho_L - \rho_v}{g_0} D^3. \tag{2}$$

Forces opposing bubble departure will be taken as negative. Since width of the bubble neck is very small no correction is applied to the buoyant force to account for the fact that the base area does not have the liquid pressure acting underneath it. For the same reason, the excess pressure force on the spherical surface area directly over the base have been neglected.

Surface tension force

Vertical component of the surface tension force is given by

$$F_{sy} = -\sigma\pi D_b \cos\gamma. \quad (3)$$

If geometry of the neck is known then, $D_b \cos\gamma$ can be expressed as

$$D_b \cos\gamma = G(\theta)D \quad (4)$$

where $G(\theta)$ is related to geometry of the bubble neck. The left side of equation (4) depends, through D_b and γ, on the conditions at the heating surface. Consequently, the surface conditions will influence both the neck geometry and the bubble diameter. From equations (3) and (4) one obtains

$$F_{sy} = -\sigma\pi G(\theta)D. \quad (5)$$

The explicit form of $G(\theta)$ cannot be written without knowing the neck geometry and conditions at the heating surface.

Liquid inertia force

The inertial force of the apparent liquid mass surrounding the bubble is given in [3]. Substituting the growth law given by equation (1), this force has been determined as

$$F_L = \frac{11}{384}\frac{\pi\rho_L}{g_0}\beta^4. \quad (6)$$

The liquid inertia force tends to pull the bubble off the heated surface.

Pressure restraining force (form drag)

This force is due to the specific pressure distribution on the bubble surface which tends to flatten the bubble also holds it against the wall. Witze *et al.* [4] have shown that the pressure restraining force F_R influences the growth process. For a potential flow pattern surrounding a spherical bubble which grows on a plane surface F_R is given, in [4], by

$$F_R = -0{\cdot}0181\,\pi\frac{\rho_L}{g_0}\beta^4. \quad (7)$$

It should be noted that the growth constant β is defined differently here and in [4]. This explains the different numerical constants in the expressions given for the pressure restraining force.

The pressure restraining force has not been considered in any of the earlier attempts to predict bubble departure diameters. However, this force turns out to be of the same order of magnitude as the liquid inertia force. Just before departure, the boundary layer around a bubble may be assumed to be very thin. Therefore, equation (7) has been used here without any modification.

By using the foregoing force expressions, at the moment of bubble departure, the force balance equation can be written as

$$aD_d^3 + bD_d + c = 0 \quad (8)$$

where,

$$a = \frac{1}{6}\frac{g}{g_0}(\rho_L - \rho_v) \quad (9)$$

$$b = -\sigma G(\theta)_d \quad (10)$$

$$c = 0{\cdot}0105\frac{\rho_L}{g_0}\beta^4. \quad (11)$$

$G(\theta)_d$ is the value of function $G(\theta)$ at departure.

3. SOLUTION OF EQUATION (8)

Following [5], roots of this equation may be expressed in the form:

$$[D_d]_i = -\frac{c}{b}f_i(\phi), \quad i = 1,2,3 \quad (12)$$

where

$$\phi = \frac{ac^2}{b^3}. \quad (13)$$

It is seen from the foregoing equations that

$$a > 0, b < 0 \text{ and } c > 0. \quad (14)$$

Hence, $\phi < 0$. It is required that D_d be real and positive. Therefore, $f_i(\phi)$ in equation (12) should be positive.

Within the range $-4/27 \leqslant \phi \leqslant 0$ all roots of equation (8) are real and when $\phi < -4/27$, there is one real root. For the latter, however, the real roots are negative. Therefore, for the present problem ϕ can assume values within the range

$$-4/27 \leqslant \phi \leqslant 0. \tag{15}$$

In this range there are two positive roots for each ϕ. These roots have been listed in [5] as $f_1(\phi)$ and $f_2(\phi)$. Study of the listed values indicates that only $f_2(\phi)$ satisfies the conditions of the present problem. To explain this, variations of $f_1(\phi)$ and $f_2(\phi)$ should be considered. It is evident from equation (13) that the absolute value of ϕ is proportional with the ratio of the forces which pull the bubble away from the heated surface to that which keeps the bubble attached to the surface.

As the pulling forces increase relative to the opposing force, i.e. as $|\phi|$ increases, departure diameter should decrease. Referring to equation (9), for a given fluid, coefficient (a) does not change too much with changes in temperature and pressure. Therefore, an increase in $|\phi|$ is caused by an increase in c/b ratio. Consequently, as c/b ratio or $|\phi|$ increases $f_i(\phi)$, in equation (12), should decrease for a reduction in the departure diameter. As a result, for the solution, $f_i(\phi)$ should be a monotonically decreasing function of $|\phi|$. This requirement is satisfied by $f_2(\phi)$ given in [5]. Since $G(\theta)$ is not specified, it would be convenient to express the departure diameter in the following form which is obtained by combining equations (9), (12) and (13):

$$D_d^3 = -\frac{6g_0}{g(\rho_L - \rho_v)} c\phi[f_2(\phi)]^3 \tag{16}$$

where, $c = c(\beta)$. Because of ϕ, equation (16) is still dependent on the value of $G(\theta)$ at departure. This implies that the departure diameter is influenced, in general, by localized conditions at the heating surface. For a given β, departure size D_d can be determined from equation (16) provided that the value of ϕ is known at departure. Since this requires specific information concerning the neck geometry and the surface conditions, problem becomes rather complicated. For a special case, however, the solution turns out to be very simple. Now this case will be considered.

4. MINIMUM DEPARTURE SIZE

Referring to equation (16), for a given growth parameter β, departure size has a minimum at $\phi = -4/27$ for the range specified by expression (15), with

$$f_2(\phi) = 1{\cdot}50 \text{ at } \phi = -4/27. \tag{17}$$

Considering two of the most commonly used equations for the bubble growth, parameter β is given by the Fritz–Ende [6] expression as

$$\beta = 4\left[\frac{\alpha}{\pi}\right]^{\frac{1}{2}} N_{Ja} \tag{18}$$

and by the Plesset–Zwick [7] expression as

$$\beta = 4\left[\frac{3\alpha}{\pi}\right]^{\frac{1}{2}} N_{Ja}. \tag{19}$$

Cole and Shulman [8] experimentally investigated the bubble growth rates at high Jakob numbers. They concluded that for Jakob numbers above 100, the discrepancy between existing bubble growth theories and experiment becomes increasingly greater. These investigators recommend the following empirical relation

$$\beta = 5(\alpha)^{\frac{1}{2}} N_{Ja}^{\frac{3}{4}}. \tag{20}$$

Now, the minimum bubble departure diameter, D_{min}, can be determined by substituting equations (11) and (17) in equation (16). Parameter β is obtained from one of the equations (18)–(20). Consequently, the dimensionless minimum departure diameter, $\bar{D}_{min}$, is given by one of the following three equations which are based, respectively, on equations (18)–(20):

$$\bar{D}_{min} = D_{min}[g/\alpha^2]^{\frac{1}{3}} = 0{\cdot}935\, N_{Ja}^{\frac{4}{3}} \tag{21}$$

$$\bar{D}_{min} = 1{\cdot}95\, N_{Ja}^{\frac{4}{3}} \tag{22}$$

$$\bar{D}_{min} = 2{\cdot}7\, N_{Ja}. \qquad (23)$$

In deriving these equations it has been assumed that $\rho_L - \rho_v \simeq \rho_L$.

The foregoing equations indicate that the minimum departure diameter is function of Jakob number only.

5. COMPARISON WITH EXPERIMENTS AND DISCUSSION

The foregoing three equations for $\bar{D}_{min}$ have been plotted in Fig. 2. The smallest experimental departure diameters reported by several investigators are also shown in the same figure. It is reasonable to assume that most of these expermental values are not the real minimums although they are the smallest departure diameters reported for different experimental conditions.

So far as the minimum departure size is concerned, it is seen that especially at Jakob numbers larger than 100 the best agreement with the experimental data is given by equation (23). Some of the reported data is seen to be somewhat smaller than the minimum sizes predicted by this equation. However, it should be noted that the growth parameter β used in equation (20) and consequently in equation (23) was given in [8] as an average value. It has been reported in [2] that the spherical bubbles have the smallest growth rates. If equation (20) which was obtained from Fig. 5 of [8] is determined to correlate the smallest growth constant values given in this figure then, the predicted minimum departure size will be represented in Fig. 2 with the broken straight line. In this case, it is seen that all experimental departure diameters lie above this line. However, it is believed that equation (23), as an approximation, represents variation of the minimum departure diameter reasonably well.

Next, a brief discussion seems to be in order which concerns the departure diameter under general conditions. Referring to equation (16), it is seen that ϕ is not known and its prediction is not an easy matter. Because of the lack of this

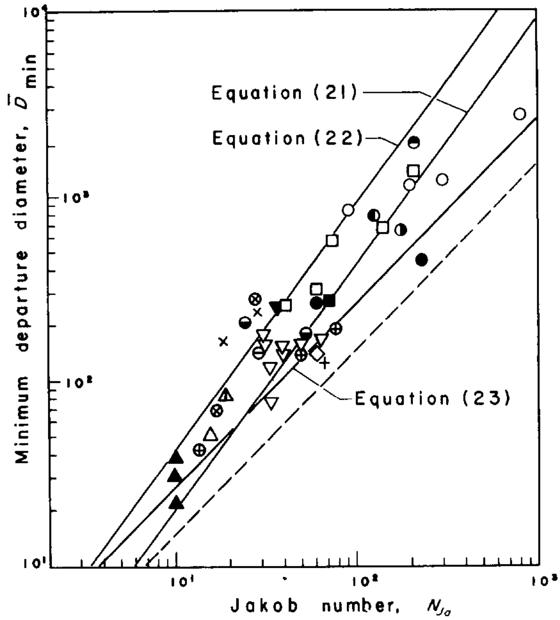

FIG. 2. Variation of the dimensionless minimum departure diameter vs. the Jakob number.

Data source
⊗ Fritz and Ende [6] Water 1 atm
○ Cole and Shulman [8] Water, pressure: 50–360 mm Hg
◑ Cole and Shulman [8] Carbon tetrachloride, 138 mm Hg
◐ Cole and Shulman [8] Acetone, 222 mm Hg
□ Cole and Shulman [8] Methanol, 134–540 mm Hg
◒ Cole and Shulman [8] Toluene, 48 mm Hg
◓ Cole and Shulman [8] n-pentane, 524–760 mm Hg
+ Gaertner and Westwater [10] Aqueous solution of nickel salts, 1 atm
▲ McFadden and Grassmann [11] Liquid nitrogen, 1 atm
⌵ Van Stralen [12] Water, 1 atm
× Siegel and Keshock [13] Water, 1 atm
▼ Perkins and Westwater [14] Methanol, 1 atm
⊕ Hatton and Hall [15] Water, 4·47–14·7 psia
△ Zmola [16] Water, 1 atm
◬ Howell and Siegel [17] Water, 1 atm
⊖ Han and Griffith [18] Water, 1 atm
▽ Gaertner [19] Water, 1 atm
■ Westwater and Santangelo [20] Methanol, 1 atm
● Raben *et al.* [21] Water, 10–760 mm Hg

information, the departure diameter cannot be determined. Nevertheless, results of the force balance method provide the relevant departure parameters which might be used for correlating the experimental data.

The departure diameter has been obtained earlier by eliminating b among equations (9), (12) and (13). Instead of this if c is eliminated then,

$$D_d^2 = -6\,\phi f_2^2(\phi) G(\theta)_d \left[\frac{g_0 \sigma}{\rho_L - \rho_v}\right] \quad (24)$$

where $\phi < 0$. Referring to equations (13), (16) and (24) it is evident that the departure diameter is influenced by three parameters: $G(\theta)$, N_{Ja}, and $[g_0\sigma/(\rho_L - \rho_v)]$. Consequently, an attempt to correlate the experimental departure sizes in terms of these three parameters is reasonable. Such a correlation has been proposed by Cole [9]. This author has chosen $G(\theta)$ to be directly proportional to the contact angle θ. A more recent correlation proposed by Cole and Rohsenow [22] uses a modified Jakob number which does not involve the wall superheat and treats the contact angle as a constant. This, of course, simplifies the correlation. However, the authors were careful to note that the deviation of the experimental data from the proposed equations was partly a result of neglecting the effects of the dynamic contact angle and probably the wall superheat.

It has become evident that the balance of forces technique, although useful, is limited in scope since, this approach can not take into account fully effects of the induced fluid motion on the bubble growth and departure process. There is, however, increasing evidence that these effects are important and cannot be neglected. Development of a general departure criterion based on the flow instabilities especially seems promising. This is one reason of the inclusion of a small neck in the bubble model of the present analysis.

In the case of film boiling from a horizontal surface, the author showed that [23] the maximum departure diameter could be determined by considering the instability characteristics of the bubble neck. It was shown that the departure would occur as soon as the bubble neck became unstable. This approach is being used by the author to study the departure process in the case of nucleate pool boiling.

For low viscosity liquids, the friction drag force can be neglected in the force balance approach; even at high Jakob numbers. However, the viscous effects can change noticeably the flow conditions around and inside the growing bubble and thus influence the growth and the departure process. Unfortunately, details of the fluid flow field cannot be incorporated into the force balance equation.

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DIAMÈTRE MINIMUM DE BULLE AU DÉPART LORS D'UNE ÉBULLITION NUCLÉE EN RÉSERVOIR

Résumé—On décrit une méthode pour déterminer le diamètre minimum de bulle au départ dans une ébullition nuclée saturée en réservoir. On montre que la taille minimum au départ varie seulement avec le nombre de Jakob. Les résultats sont comparés avec les plus petits diamètres de départ obtenus expérimentalement et rapportés dans la littérature. Un accord satisfaisant est obtenu quand la taille minimum a été déterminée à partir de la loi de vitesse de croissance de la bulle recommandée par Cole et Shulman·

DER MINIMALE BLASENABREISSDURCHMESSER FÜR BLASENSIEDEN BEI FREIER KONVEKTION

Zusammenfassung—In dieser Arbeit wird eine Methode zur Bestimmung des minimalen Blasenabreissdurchmessers für Blasensieden bei freier Konvektion und Sättigung beschrieben. Es zeigt sich, dass die minimale Abreissdurchmessergrösse nur mit der Jakob-Zahl variiert. Die berechneten Ergebnisse werden mit den kleinsten experimentell bestimmten Abreissdurchmessern aus der Literatur verglichen, Eine befriedigende Übereinstimmung wird erzielt, wenn die minimale Blasenabreissdurchmessergrösse auf der Basis des Blasenwachstumsgesetzes von Cole und Shulman bestimmt wird.

О МИНИМАЛЬНОМ ДИАМЕТРЕ ОТРЫВА ПУЗЫРЬКА ПРИ ПУЗЫРЬКОВОМ КИПЕНИИ В БОЛЬШОМ ОБЪЁМЕ

Аннотация—Описывается метод определения минимального диаметра отрыва пузырька при пузырьковом кипении в большом объёме. Показано, что минимальный размер отрыва пузырька изменяется только в зависимрсти от числа Якоба. Результаты расуётов сравниваются с наименьшими диаметрами отрыва, наблюдающимися экспериментально и писанными в литературе. Наблюдается удовлетворительное соответствие с минимальным размером отрыва, полученным на основе закона скорости роста, рекомендуемого Коулом и Шульманом.